Project title: Generating functor calculi

Supervisor(s):	**David Barnes, Niall Taggart**
Helpful existing knowledge:	**algebra, topology**

Project Description:

Functors provide a systematic way of transforming one mathematical object into another. For example, homology is a functor from the category of simplicial complexes to the category of abelian groups. In many respects, functors can be viewed as an abstraction, or *categorification*, of functions, and *functor calculus* extends this analogy by developing a categorical version of Taylor's theorem from classical analysis.

For suitably well-behaved functors, a functor calculus constructs a sequence of "polynomial" approximations, with the precise notion of polynomial depending on the context. This analogy with Taylor expansion has become a central idea in modern algebraic topology, influencing a wide range of research directions. However, constructing new instances of functor calculi remains a subtle and challenging task.

A powerful way to understand a functor calculus is to move one step further in abstraction and study the *category of all polynomial functors* rather than individual approximations. These categories often have elegant descriptions that reveal deep structural information about the calculus itself.

Recent work by the supervisors and their collaborators has demonstrated that this perspective can lead to significant new insights. The aim of this PhD project is to build on these developments by developing a unified framework and generating new examples of functor calculi. The range of potential examples is broad and can be guided by the student's interests, with possible directions including purely algebraic settings, applications in topological data analysis, and connections to differential topology.

The student should have attended courses on both topology and algebra. This project will require the student to become familiar with the abstract language of infinity categories and stable homotopy theory.

Useful references

Arone, G. and Ching, M., "Goodwillie calculus." *Handbook of Homotopy Theory* (2019).

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Barnes, D., Kędziorek, M., & Taggart, N., "A Dwyer-Rezk classification for polynomial functors in Weiss calculus." https://arxiv.org/abs/2508.03808 (2025).

Barnes, D., and Roitzheim, C. "Foundations of stable homotopy theory". *Vol. 185. Cambridge University Press* (2020).

Cnossen, B., "Stable homotopy theory and higher algebra" *available from the authors website https://sites.google.com/view/bastiaan-cnossen/home* (2025)

Johnson, B., & McCarthy, R., "Deriving calculus with cotriples." *Transactions of the American Mathematical Society* (2003).

Land, M., "Introduction to infinity-categories". *Springer Nature* (2021).